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Analysis of Bullet and Ball Collision using Theoretical Approach

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Abstract

Collisions between two objects can be analyzed using the concept of the law of momentum conservation. In the case of a collision between two objects, namely a bullet and a ball, it is necessary to analyze using a theoretical approach. This research aims to analyze the velocity and height after a collision. Newton's law of conservation of momentum and Newton's second law in friction conditions are used as the basis of Calculation analysis using analytical and numerical methods. There are air friction coefficients with variations of 0.1, 0.2, 0.3, 0.4, and 0.5 N.s/m. This type of bullet used a 9 mm caliber with a velocity of 380 m/s. The mass of the bullet and ball is 0.008 kg and 0.3 kg, respectively. The results showed that increasing the air fraction coefficient resulted in lower velocity and height achieved by bullet and ball. The increasing discretization value makes the results of numerical calculations consistent with analytical calculations

INTRODUCTION

Physics is a branch of natural science that studies and explains the natural phenomenon and the interactions between objects in the universe [1,2]. A collision between two objects is a natural phenomenon that can be studied and analyzed using the concept of momentum [3]. A technology is needed to support the collision motion analysis between two objects and prove the law of momentum conservation. The technology that can be used is the tracker video analysis [4]. Tracker video analysis is built using the Open Source Physics (OSP) Java framework that can be run on the Windows operating system to support various experimental activities in physics education [3,5,6].

The analysis results can be displayed in images, tables, and graphs [7,8]. Tracker video analysis has been used to determine the coefficient of restitution for the collision of marbles with double hangers, and the resulting error value is lower than the commercial air cushion model [4]. In addition, tracker video analysis can also analyze the collision of tennis balls and ping pong balls against the ceramic floor to get the coefficient of restitution even though the camera is not precise enough to track the ball's position at any time [9].

The bullet collision experiment with the ball, as shown in Fig 1, cannot be carried out directly using tracker video analysis because the experiment must be carried out in a standardized ballistics test laboratory, such as the research conducted by Nazarudin [10] to test glass-fiber reinforced polyester (GRP) composite against projectile shots. The correct tracking process is done by means that a camera must support it with adequate resolution and framerate per second (fps) [9].

An alternative that can be used is to make a ballistic pendulum using a microcontroller for inelastic collisions by Rohman and Dzulkiflih [11]. The ballistic pendulum uses two sensors, namely a rotary encoder sensor to measure the angle of the pendulum when the bullet collides with the pendulum and a photodiode sensor to measure the velocity of the bullet before it collides with the pendulum [11]. The experimental results only measure the velocity and momentum before and after the collision. Then Salimi *et al.* [12] used the LS-Dayna software to simulate bullet projectile collisions with ceramic based on variations in the firing angle. The simulation results provide information on the graphical data of the bullet velocity after collision with ceramics and a simulated image of when the bullet is fired at different angles.

Based on previous research, the study of bullet collisions with the ball and other objects is still being carried out experimentally and using software for simulation. This research analytically studies the collision of bullets with a ball using the law of momentum conservation and Newton's second law with the effect of air friction to obtain the equation for velocity and height after the collision. In addition, a numerical calculation has been carried out to compare the result of the analytical calculation.

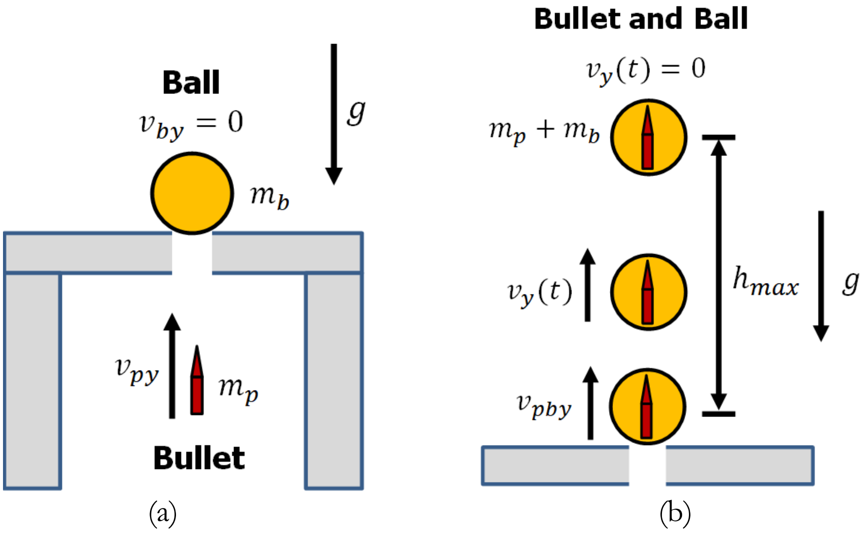

Figure 1. Schematic of the ball and bullet collision (a) before and (b) after the collision.

RESEARCH METHODS

The bullet and ball collision shown in Fig 1 is entirely inelastic. Before the bullet collides with the ball, the air friction on the bullet is negligible. After the collision, there is an effect of air friction when the bullet and ball move up (Fig 1(b)). In this research, the type of bullet model is 9 mm caliber. The mass and bullet velocities were 8 grams and 380 m/s, respectively [10]. The ball used as the target is assumed to have a uniform density with a mass of 300 grams. The air friction coefficient values are varied, namely 0.1, 0.2, 0.3, 0.4, and 0.5 N.s/m. Earth's gravitational acceleration is 9,8 m/s^2. The analysis process was carried out using two methods, namely analytical methods and numerical methods. Hasil analisis berupa data kecepatan, dan tinggi peluru dan bola.

RESULTS AND DISCUSSION

a. Equation of Bullet and Ball Velocity after Collision

In Figure 1(a), the bullet is not affected by air friction, and when the bullet collides with the ball, as shown in Figure 1(b), the equation for the bullet and ball velocity after the collision using the law of momentum conservation can be written as follows:

$$p_y = p'_y \tag{1}$$

$$m_p v_{py} + m_b v_{by} = (m_p + m_b) v'_y \tag{2}$$

Before the collision, the ball is at rest, so $v_{by} = 0$ and v'_y becomes v'_{pby}. The equation for the bullet and ball velocity after a collision is

$$v_{pby} = \left(\frac{m_p}{m_p + m_b} \right) v_{py} \tag{3}$$

Where v_{pby} is the bullet and ball velocity after collision in the vertical direction (m/s), v_{py} is bullet velocity before collision (m/s), m_p is bullet mass (kg), and m_b is ball mass (kg)

b. Analytical Calculation of the Bullet and Ball Motion with Air Friction

The following analysis is an analytical calculation of the effect of air friction when the bullet and ball are in the air (Figure 2).

$$f_y = -\alpha v_y \tag{4}$$

With f_y its air friction force (N), α is air friction coefficient (N.s/m), and v_y is bullet and ball velocity as it moves up (m/s).

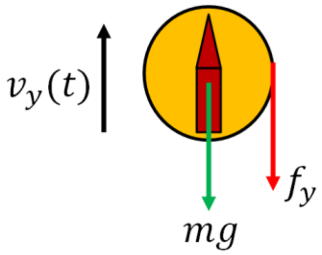

Figure 2. The free force diagram of the bullet and ball when it is in the air with the effect of air friction.

Based on Figure 2, the bullet and ball motion equation can be found using Newton's second law. The equation can be written as follows:

$$m\frac{d^2y}{dt^2} = -mg - \alpha v_y \tag{5}$$

From the derivation of equation (5), the equation for velocity and height is given by:

$$v_y(t) = v_{0y}e^{-\frac{\alpha}{m}t} + \frac{mg}{\alpha}\left(e^{-\frac{\alpha}{m}t} - 1\right) \tag{6}$$

$$y(t) = \frac{m}{\alpha}\left(v_{0y} + \frac{mg}{\alpha}\right)\left(1 - e^{-\frac{\alpha}{m}t}\right) - \frac{mg}{\alpha}t \tag{7}$$

Equations (6) and (7) have similarities with the equations derived by Tajima and Fujisawa [13] in analyzing the projectile trajectory of penguins' feces. Although it produces the same equation, the different variables are m and v_{0y}. In this paper, m mass is total $(m_p + m_b)$ while v_{0y} is the bullet and ball velocity after collision (v_{pby}). The calculation result of velocity and height against time can be seen in Figure. 3.

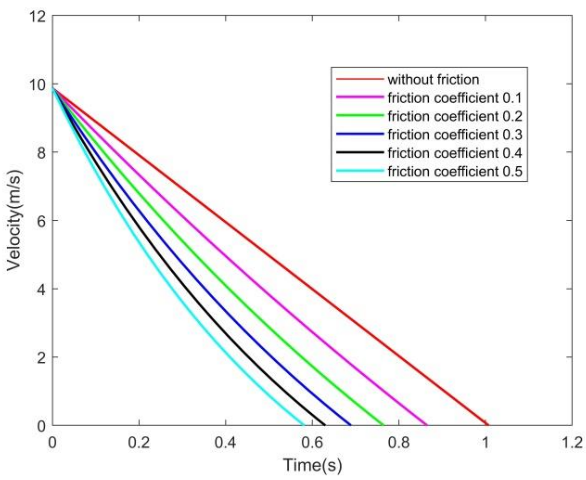

Figure 3. Velocity

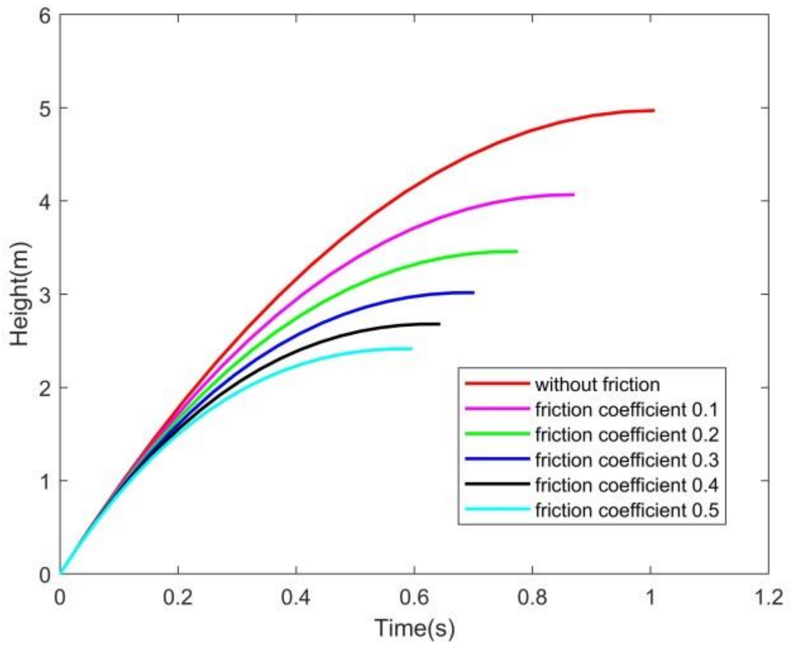

Figure 4. Height for each air resistance coefficient.

In Figure 3, the increase in air friction coefficient makes the bullet and ball velocity becomes smaller. As a result, the time for the bullet and ball to move upward becomes shorter. Table 1 shows that the maximum height decreases with increasing the air friction coefficient because the air friction force reduces velocity when the bullet and ball move up.

Table 1. The maximum height of the bullet and ball for each air resistance coefficient.

v_p (m/s) [10]	m_p (kg) [10]	m_b (kg)	α (N.s/m)	h_{max} (m)
380	0,008	0,3	0,1	4,07
380	0,008	0,3	0,2	3,46
380	0,008	0,3	0,3	3,01
380	0,008	0,3	0,4	2,68
380	0,008	0,3	0,5	2,41

c. Numerical Calculation of Bullet and Ball Collision with Air Friction

The second-order differential equation in equation (5) is quite difficult to solve analytically, so an approach with numerical calculations is needed. The methods commonly used in the numerical calculation are the finite difference method, the finite element method, the finite volume method, and others. This technique is used to solve relatively complex physical problems such as fluid flow in porous media [12,13], heat transfer distribution in materials [16], and even in finance and industry [17]. In this research, the finite difference method will be used because this method is stable enough for less complex cases and is more efficient. The finite difference method is a numerical solution that utilizes domain discretization based on a Taylor series with boundary conditions along the edge of the domain.

The boundary conditions in this problem are the initial velocity and height, so the application uses an explicit scheme. In this scheme, the quantity at a point was calculated directly from the previous quantity [18]. The form of the Taylor series expansion used to approximate the differential equation is as follows:

$$y(t+\Delta t)=y(t)+\frac{\Delta t}{1!}y'(x)+\frac{(\Delta t)^2}{2!}y''(x)+\frac{(\Delta t)^3}{3!}y'''(x)+\ldots$$
$$y(t-\Delta t)=y(t)-\frac{\Delta t}{1!}y'(x)+\frac{(\Delta t)^2}{2!}y''(x)-\frac{(\Delta t)^3}{3!}y'''(x)+\ldots \quad (8)$$

Where Δt is the time interval between two adjacent vertices, In the motion equation of Newton's second law, the equation for the bullet and ball velocity can be written as follows:

$$\frac{dv_i}{dt}=-g-\frac{\alpha}{(m_p+m_b)}v_i \quad (9)$$

After applying the finite difference method with the forward scheme in equation (9), this equation can be rewritten as:

$$\frac{(v_{i+1}-v_i)}{\Delta t}=-g-\frac{\alpha}{(m_p+m_b)}v_i$$
$$v_{i+1}=-g\Delta t-\left(1-\frac{\alpha\Delta t}{(m_p+m_b)}\right)v_i \quad (10)$$

The exact process can be used to get the height of the bullet and ball equation as follows:

$$y_{i+1}=y_i+y_i'\Delta t+\left(-g-\frac{\alpha}{(m_p+m_b)}y_i'\right)\frac{(\Delta t)^2}{2} \quad (11)$$

The idea of numerical computation is to divide the problem domain into smaller parts. The boundary conditions used in this problem are $y(0)=0$ and $v(0)=v_{py}$. The results of calculating velocity and height using analytical and numerical methods can be seen in Figure 4.

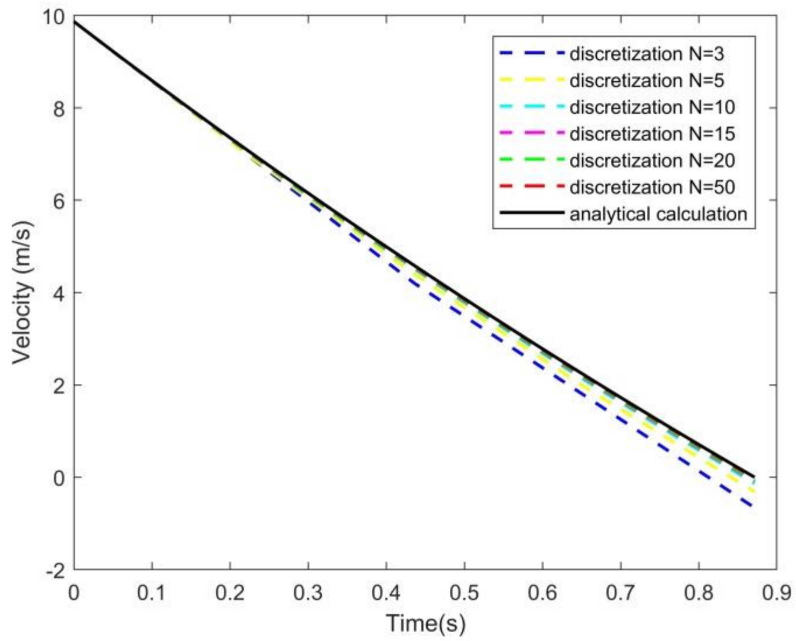

Figure 4. Comparison of analytical and numerical calculation with different discretization values for the case-velocity.

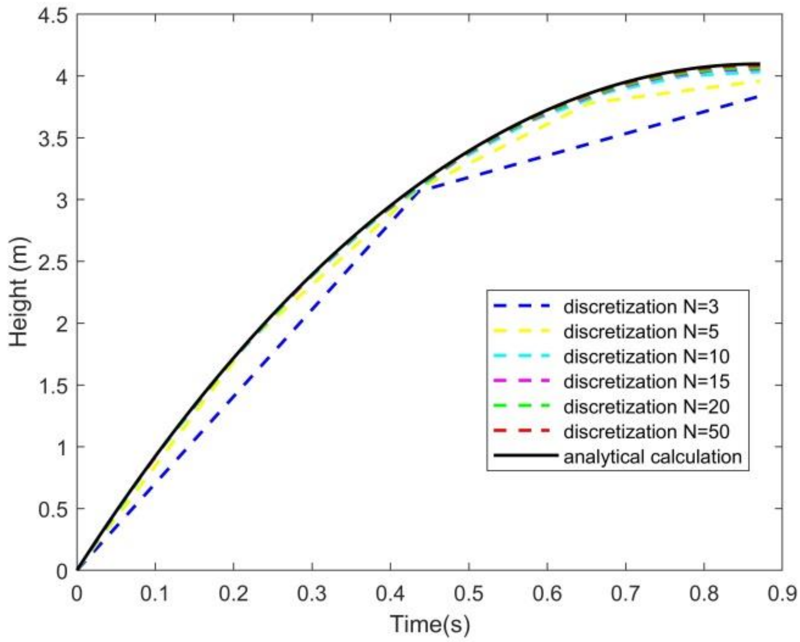

Figure 5. Comparison of analytical and numerical calculation with different discretization values for the case-height.

Based on Figure 4 anda 5, it can be seen that the more excellent the discretization value, the results of numerical calculations using the finite difference method will approach the analytical calculation result because the finite difference method utilizes a domain discretization based on a Taylor series with boundary conditions along the domain edge [19]. Data discretization makes the calculation results easier to understand [20]. These results indicate that numerical calculations using the finite difference method can be an alternative method for determining velocity and height.

CONCLUSION

Using a theoretical approach, analysis of bullet and ball collisions has succeeded in obtaining velocity and height data with analytical and numerical calculations. Increasing the value of the air friction coefficient reduces the velocity and height of the bullet and ball. Numerical calculations using the explicit scheme finite difference method can be an alternative in determining the velocity and height based on discretization number.

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